Pluralism of Mathematical Understanding – A Process-Based Approach

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Some Statements about Mathematics

- $1 + 1 = 2$
- $2 \times 3 = 1 \times 6$
- $\pi > 3$
- Every natural number has a successor.
- The consistency of arithmetic needs a proof to be true.
- Any total function defined in $[r_0, r_1](r_0 < r_1)$ is uniformly continuous.

True, or not?

- $1 + 1 = 2$
- $2 \times 3 = 1 \times 6$
- $\pi > 3$
- Every natural number has a successor.

- The consistency of arithmetic needs a proof to be true. (?)
- Any total function defined in $[r_0, r_1](r_0 < r_1)$ is uniformly continuous. (?!)

Disagreement among Mathematicians

- The consistency of arithmetic needs a proof to be true. (?)
  - Classical mathematicians – Yes! (Gentzen's proof in 1936)
  - Pre-intuitionistic mathematicians (Poincaré, Borel, Lebesgue) – No!
- Any total function defined in $[r_0, r_1](r_0 < r_1)$ is uniformly continuous. (?!)
  - Classical mathematicians – No! Not even continuous.
  - Intuitionistic mathematicians (Brouwer) – Yes! (the viscous reals)

What does the disagreement mean?

What is indicated by the disagreement in truth-evaluations of cognitively significant mathematical propositions?

- A broken concept of mathematics?
- Different understanding of mathematics?

⇒ Initial motivation of my dissertation:

Explain the disagreement without evaluating the rationality of different mathematical schools

♡ A possible answer:

A process-based account that entails a pluralistic stance on mathematical understanding

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Why thematize understanding?

What is indicated by different truth-evaluations of cognitively significant mathematical propositions?

- A broken concept of Mathematics? – NO...

  It is almost an inference from cognitive level to factual level.

- Different understanding of mathematics? – YES!!
  1. Understanding is considered an epistemic goal;
  2. the success of understanding can be evaluated relative to agents

On Understanding and Understandability (I)

$\Rightarrow$ '*S* understands *O*' and '*O* is understandable to *S*'

The truth value of '*S* understands *O*' varies across agents.[1]

- **Irrelevant**: my successful understanding of $2 \times 3 = 1 \times 6$ & whether my neighbor understands it
- **Not necessarily relevant**: my current successful understanding of $2 \times 3 = 1 \times 6$ & my inability of understanding it when I was two-year old

[1] *S* refers to an understanding agent, *O* to an object in the domain of mathematics e.g. a proposition, a concept, a proof.

On Understanding and Understandability (II)

$\Rightarrow$ '*S* understands *O*' and '*O* is understandable to *S*'

It's fine to assume optimistically: The truth value of '*O* is understandable to *S*' is absolute, for all agents with human-alike cognitive abilities.

- $2 \times 3 = 1 \times 6$ is **understandable** to me and my neighbor, even to an alien with human-alike cognitive abilities.
- $2 \times 3 = 1 \times 6$ is **understandable** to me (now and then).

Payoffs of Involving Understandability (I)

1. A legitimately basis of talking about:
   O – an understood object in the domain of mathematics
   S – an understanding agent S with human-alike cognitive abilities

2. A way to transform from particular mathematical understanding to its general form
   - by treating 'the pair (S, O)' as the common structure of the subject of the present project, meaning **any** S and **any** O fall under the understandability relation
   - by studying the **stimulus conditions** that allow some S and some O falling under the understanding relation

Payoffs of Involving Understandability (II)

3. An inherited dependence relation for free, by treating
   - the relational property, understandability, as a **disposition** of the pair (S, O)
   - the relational property, understanding, as a **manifestation** of this disposition
   - mathematical practices exercised by S as a **necessary part of the stimulus conditions** for the manifestation of this disposition

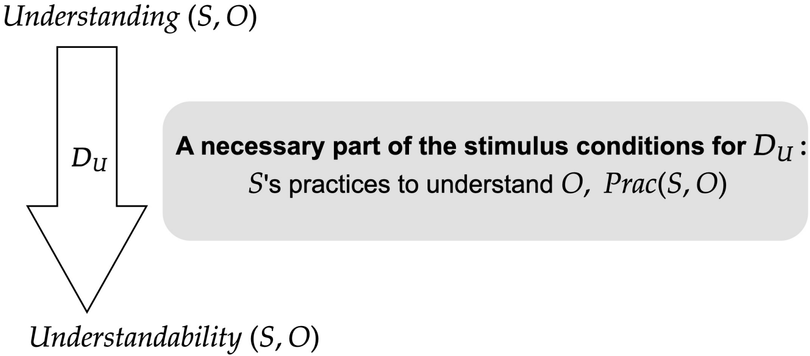


Figure: The Inherited Dependence

Why analyze mathematical practices?

Fact: Mathematical understanding arises from mathematical practices and the practices of mathematical agents differ from case to case.

- The analyses of real-world mathematical practices allows us to study the disagreement of the truth-evaluation of mathematical proposition via a more **fine-grained** study of mathematical understanding.
- Since it **localizes the contextual influences** on mathematical understanding at the practical, non-cognitive level, analyzing mathematical practices prevents us from making cognitive evaluation about the rationality of mathematical agents.

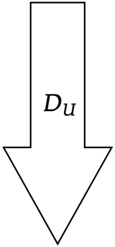
What is analyzed?

Actions, objects and the use of cognitive abilities (read functionally):

- The **actions** performed by *S* to understand *O* – their constitutive role in *S*'s understanding of *O*.
- The **objects** that *S* accepts in order to act on them to understand *O* – the informational items they carry that can be mutually related to provide content for *S*'s understanding of *O*.
- *S*'s **use of cognitive abilities** for understanding *O* – the constitution of *S*'s understanding of *O*.

$Understanding\ (S, O)$ – constitution of understanding in terms of ability use

$Constitution(Understanding\ (S, O))$



A necessary part of the stimulus conditions for D_U :

S's practices to understand O, $Prac(S, O)$

$$Prac(S, O) : \begin{cases} \text{actions (constitutive roles)} & C(Prac(S, O)) \\ \text{objects (informational roles)} & I(Prac(S, O)) \end{cases}$$

$Understandability\ (S, O)$

Figure: The Structure of Strategy

Questions and Comments, Please!
THANK YOU!!