



# É. Cartan, fonds 38J, Archives de l'Académie des Sciences

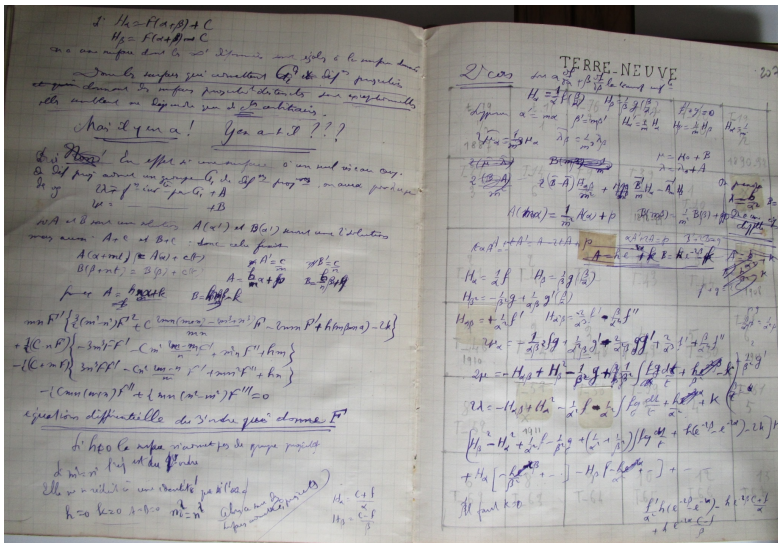


Figure: Cartan 38J, 1-31, p. 206-207, Surfaces admettant un groupe continu  $G_2$  de déformations projectives. (ca. 1927-28)

# The genesis of Dedekind's *Dualgruppen*

$a = [1, \omega]$	$\alpha' = [2, \omega]$	$b' - r' = [1, \omega]$	$a_1 = [6, 2+5\omega]$	$b_1 + r_1 = [2, \omega] = a - \alpha'$
$b = [2, \omega]$	$b' = [1, \omega]$	$r' - \alpha' = [2, \omega]$	$b_1 = [6, 2+5\omega]$	$r_1 + \alpha_1 = [2, \omega] = b - b'$
$r = [6, 2+5\omega]$	$r' = [1, \omega]$	$\alpha' - b' = [2, \omega]$	$r_1 = [2, \omega]$	$\alpha_1 + b_1 = [6, 2+5\omega] = r - r'$

$(a - \alpha', b_1 + r_1) = (a - (b+r), (r-a) + (a-b)) = (a - (b+r), a - (b+r))$

$(b' - r', a + \alpha_1) = ((r-a) - (a+b), a + (b-r))$

Die Zahl in  $a - \alpha' = a - (b+r)$  hat die Form

$x = \beta + \gamma$ ; da hierin  $\beta = x - \gamma$ ,  $\gamma = x - \beta$  folgt

so ist  $\beta$  in  $b - (a+r) = b - b'$

$\gamma$  in  $r - (a+b) = r - r'$

d.h.

$\left. \begin{aligned} a - \alpha' &> (b - b') + (r - r') \\ b - b' &> (r - r') + (a - \alpha') \\ r - r' &> (a - \alpha') + (b - b') \end{aligned} \right\}$

$(a - \alpha') + (b - b') > (b - b') + (r - r')$  folgen daraus  $\beta > \gamma$

$a + \alpha_1 < (b + b_1) - (r + r_1)$

$\left. \begin{aligned} \beta &= \beta + \gamma = \gamma + \gamma_1 \\ \beta_1 &= (\gamma) = (\alpha) \\ \gamma_1 &= (\alpha) = (\beta) \end{aligned} \right\}$

$\beta_1 = \gamma - \alpha_1$   
 $\beta_1 = \gamma - \alpha_1$

$\beta = \beta + \gamma - \alpha_1$

Allgemein  $= \alpha' - b' - r'$

$(b - b') + (r - r') = (r - r') + (a - \alpha') = (a - \alpha') + (b - b')$  mal  $b$

$(b + b_1) - (r + r_1) = (r + r_1) - (a + \alpha_1) = (a + \alpha_1) - (b + b_1)$  mal  $c$

ideale:  $= \alpha_1 + b_1 + r_1$

$a = mbc\alpha'$

$b = mca b'$

$r = ma b c'$

$b + r = a' = ma$ ;  $b - r = \alpha_1 = malc b' c'$

$r + a = b' = mb$ ;  $r - a = b_1 = malc c' a'$

$a + b = r' = mc$ ;  $a - b = r_1 = malc c' b'$

$b' - r' = mbc = a + \alpha_1$

$r' - \alpha' = mca = b + b_1$

$\alpha' - b' = ma b = r + r_1$

$b_1 + r_1 = malc a' = a - \alpha'$

$r_1 + \alpha_1 = malc b' = b - b'$

$\alpha_1 + b_1 = malc c' = r - r'$

Im Allgemeinen ist aber nur

$b' - r' < a + \alpha_1$  und  $b_1 + r_1 > a - \alpha'$

$r' - \alpha' < b + b_1$   $r_1 + \alpha_1 > b - b'$

$\alpha' - b' < r + r_1$   $\alpha_1 + b_1 > r - r'$

$m = a + b + r$

$a = \frac{b+r}{a+b+r}$

$b = \frac{r+a}{a+b+r}$

$r = \frac{a+b}{a+b+r}$

$a' = \frac{a(a+b+r)}{(r+a)(a+b)}$

$b' = \frac{b(a+b+r)}{(a+b)(b+r)}$

$c' = \frac{r(a+b+r)}{(b+r)(r+a)}$

Figure: Cod. Ms. Dedekind X 11-1, p.28.

## Ongoing projects

- ▶ Brouillons mathématiques (2022-2025), Émergence(s), ville de Paris, Institut des textes et manuscrits modernes, ENS.  
<http://www.item.ens.fr/brouillons-mathematiques-projet-emergences-2022-2026/>
- ▶ PHILIUMM (2021-2026), ERC project, D. Rabouin, SPHERE, CNRS. <https://eman-archives.org/philiumm/>
- ▶ BANANA (2023-2027), ANR project, C. Eckes & E. Haffner, Archives Henri Poincaré & ITEM.  
<http://www.item.ens.fr/banana>
- ▶ To appear: Sciences exactes, special issue of *Genesis* (2025).

## Some publications on drafts

- Haffner E (2017) Esquisse d'une cartographie des cahiers d'Élie Cartan. *Revue d'histoire des mathématiques* 23(1):125–182
- Haffner E (2018) From modules to lattices, insight into the genesis of Dedekind's Dualgruppen. *British Journal for History of Mathematics*, pp 23–42
- Haffner E (2021), The Shaping of Dedekind's Rigorous Mathematics: What Do Dedekind's Drafts Tell Us About His Ideal of Rigor?, *Notre Dame Journal of Formal Logic* 62(1), pp. 5-31
- Haffner E (2021) The edition of Bernhard Riemann's collected works: then and now. *European Mathematical Society Magazine* 120:29–39
- Haffner E (2024) Going to the Source(s) of Sources in Mathematicians' Drafts, *Research in History and Philosophy of Mathematics. The CSHPM 2022 Volume*, M. Zack et D. Waszek (éds), Birkhäuser, pp. 83-110
- Haffner E (2024) Duality as a guiding light in the genesis of Dedekind's Dualgruppen. In: Krömer R, Haffner E (eds) *Duality in 19th and 20th century mathematical thinking*, Birkhäuser, Basel
- Gentil S (2021) Une caractéristique pour les unifier toutes et dans l'harmonie les lier. Unification des équations dans les textes De la méthode de l'universalité. *Philosophia Scientiæ* 25(2):47–70
- Joffredo T (2019) Une analyse génétique de l'Introduction à l'analyse des lignes courbes algébriques de Gabriel Cramer (1750). *Revue d'Histoire des Mathématiques* 25(2):235–289
- Krömer R (2023) Duality à la Bourbaki. In: Krömer R, Haffner E (eds) *Duality in 19th and 20th century mathematical thinking*, Birkhäuser, Basel
- Remaki A (2021) L'art combinatoire en tant qu'art d'inventer chez Leibniz, sur la période 1672-1680. PhD thesis, Université de Paris, Paris